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Technical Notes

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Structure-Attached Corotational Fluid Grid for Transient Aeroelastic Computations

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Introduction

To predict the dynamic response of a rigid or flexible structure in a fluid flow, the equations of motion of the structure and the fluid must be solved simultaneously. A straightforward approach to the solution of the coupled fluid/structure dynamic equations requires moving at each time step at least the portions of the fluid grid that are close to the moving structure (see, for example, Shankar and Ide,[1] Batina,[2] and Guruswamy[3]). The aim of this Note is to present an alternative approach for the transient solution of the aeroelastic coupled problem. The governing equations of motion of the fluid are derived with respect to multiple moving frames of reference. Each of these frames is attached to a carefully selected node of the discretized structure. For a rigid aeroelastic configuration, the result is the implicit generation of a structure-attached corotational fluid grid. In this case, no single grid point needs to be explicitly moved or updated during the transient analysis, even when the structure undergoes large rigid-body motion. For flexible configurations, the corotational approach is augmented with a scheme for updating the spatial metrics that are used for computing the unsteady flow solutions, according to the deforming shape of the structure.

Formulation of the Aeroelastic Problem in Multiple Moving Frames of Reference

We focus on the solution of the two-dimensional Euler fluid flow around a moving body using a structured grid and its corresponding computational domain. The structure is represented by an airfoil that limits the domain of flow computation by its boundary Γ_s. The strong conservation law form of the two-dimensional Euler equations in Cartesian coordinates is given by

$$\frac{\partial Q}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = 0 \tag{1}$$

where

$$Q = \begin{bmatrix} \rho \\ \rho u \\ \rho v \\ e \end{bmatrix}, \quad E = \begin{bmatrix} \rho u \\ \rho u^2 + P \\ \rho u v \\ (e + P)u \end{bmatrix}, \quad F = \begin{bmatrix} \rho v \\ \rho u v \\ \rho v^2 + P \\ (e + P)v \end{bmatrix} \tag{2}$$

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ρ is the density, $V = (u, v)$ is the velocity vector, e is the total energy per unit volume and P is the pressure. The pressure P is related to the flow variable Q by the equation of state

$$P = (\gamma - 1)\,[e - \tfrac{1}{2}\rho(u^2 + v^2)] \tag{3}$$

in which γ denotes the ratio of specific heats. The flow is assumed to be uniform at infinity so that

$$\rho\infty = 1, \qquad V\infty = \begin{bmatrix} \cos\alpha \\ \sin\alpha \end{bmatrix} M\infty\, a\infty, \qquad P\infty = \frac{1}{\gamma M_\infty^2} \tag{4}$$

where α is the angle of attack and $\rho\infty$, $V\infty$, $P\infty$ and $M\infty$ are respectively the freestream density, velocity vector, pressure and Mach number. On the boundary Γ_S, the slip condition

$$V \cdot n = 0 \tag{5}$$

is enforced. For dynamic configurations, the direction of the normal n to Γ_S varies with time. Therefore, the solution of the fluid flow Eq. (1) requires the determination of the time-varying boundary $\Gamma_S = \Gamma_S(t)$. The evolution of the shape of $\Gamma_S(t)$ can be obtained through the solution of the finite-element structural dynamics equations

$$M\frac{\partial^2 q}{\partial t^2} + C\frac{\partial q}{\partial t} + Kq = f(t) \tag{6}$$

where M, C and K are respectively the mass, damping, and stiffness matrices, $q = (d_x, d_y, \theta_z)$ is the vector of nodal displacements and rotations, and $f(t)$ is the time dependent vector of prescribed nodal forces. For rigid configurations, the stiffness matrix vanishes. Clearly, Eq. (1) and (6) are coupled through the boundary condition [Eq. (5)]. The airfoil is discretized using finite elements. For simplicity, the structural mesh is designed to contain as many nodes on Γ_S as there are fluid lines of constant curvilinear coordinate. At each node j

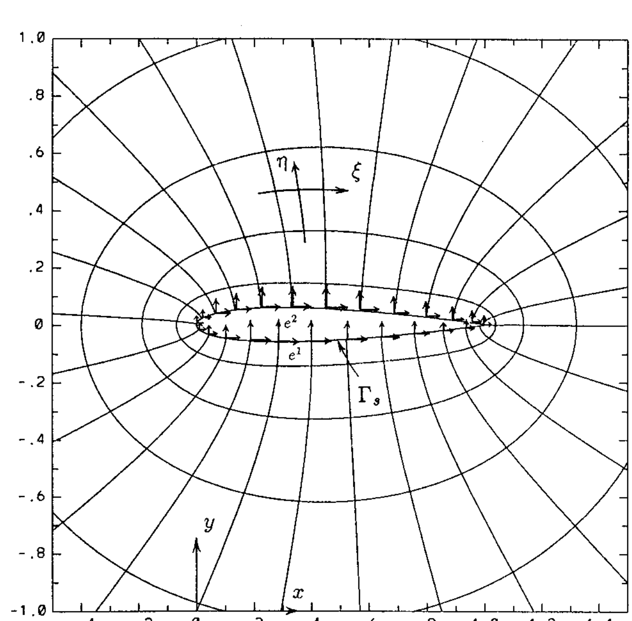

Fig. 1 Multiple moving frames of reference

on Γ_S, a time-dependent moving frame of reference $R_j(O_j(t), e_j^1(t), e_j^2(t))$ is defined: $O_j(t)$ denotes the position in an absolute frame of node j; $e_j^1(t)$ is a normalized vector that is initially parallel to c, the chord of the airfoil; and $e_j^2(t)$ is a normalized vector that is constantly orthogonal to $e_j^1(t)$ (Fig. 1).

Along the constant curvilinear coordinate curve $\xi = \xi = cte$, the conservation form of the unsteady Euler flow equations in terms of the coordinates of $R_j(O_j(t), e_j^1(t), e_j^2(t))$ is derived as

$$\frac{\partial Q'}{\partial t} + \frac{\partial E'}{\partial x'} + \frac{\partial F'}{\partial y'} = S' \tag{7}$$

where

$$Q' = \begin{bmatrix} \rho \\ \rho u' \\ \rho v' \\ e' \end{bmatrix}, \quad E' = \begin{bmatrix} \rho u' \\ \rho u'^2 + P \\ \rho u'v' \\ (e' + P)u' \end{bmatrix}$$

$$F' = \begin{bmatrix} \rho v' \\ \rho u'v' \\ \rho v'^2 + P \\ (e' + P)v' \end{bmatrix}, \quad \begin{bmatrix} S_1 \\ S_2 \\ S_3 \\ S_4 \end{bmatrix} \tag{8}$$

and

$$S_1 = -\rho \nabla \cdot V_t$$

$$S_2 = -\rho a_{tx} - \rho u' \nabla \cdot V_t$$

$$S_3 = -\rho a_{ty} - \rho v' \nabla \cdot V_t$$

$$S_4 = -\rho \{ V_j \cdot a_j + V_j \cdot (\dot{\omega}_j \times r') + V_j \cdot (\omega_j \times V') + V_j \cdot [\omega_j \times (\omega_j \times r')] + V' \cdot a_j + V' \cdot (\dot{\omega}_j \times r') + (\omega_j \times r') \cdot a_j + (\omega_j \times r') \cdot (\omega_j \times r') \} - (e' + P) \nabla \cdot V_t$$

$$V_t = V_j + \omega_j \times r'$$

$$V = V' + V_t$$

$$a_t = a_j + \dot{\omega}_j \times r' + \omega_j \times (\omega_j \times r') + 2\omega_j \times V'$$

$$a = a' + a_t$$

$$e' = e + \rho V \cdot V_t \tag{9}$$

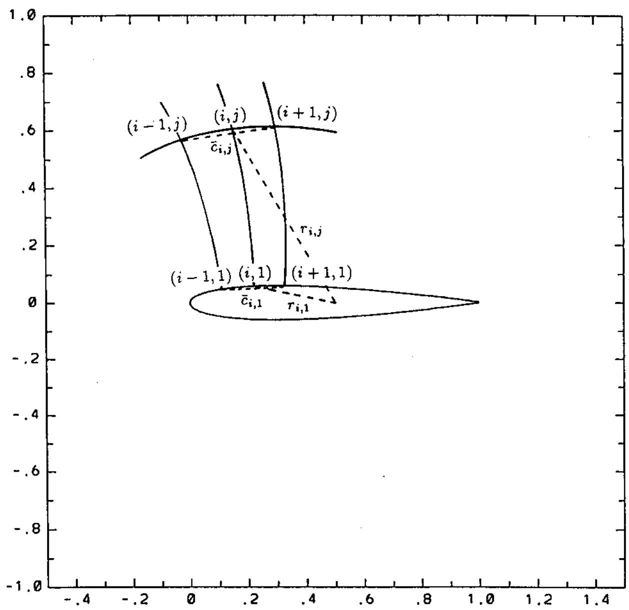

Fig. 2 Jacobian metrics updating scheme.

V and a are the flow velocity and acceleration in the Eulerian coordinates, and $V' = (u', v')$ and a' are the flow velocity and acceleration with respect to $R_j(t)$. The quantities V_j, a_j, ω_j, and $\dot{\omega}_j$ denote respectively the translational velocity and acceleration and the angular velocity and acceleration of the moving frame $R_j(t)$. The transformation velocity and acceleration are denoted by V_t *and* $a_t = (a_{tx}, a_{ty})$. The total energy and the position vector of a fluid particle with respect to the moving frame are denoted by e' and r'.

For an $N_I \times N_J$ finite-difference regular grid, N_I Euler equations similar to Eq. (7) can be written, each involving a different moving frame of reference $R_j(t)$. The motion of each of these frames can be obtained from the solution of the structural Eq. (6).

Except for the right-hand-side quantity S', Eq. (7) has the same pattern as the standard strong conservation law form of the two-dimensional Euler equations in Cartesian coordinates. Therefore, the formulation of the problem in multiple moving frames of reference can still benefit from existing fluid flow solvers such as those available in the two-dimensional implicit ARC2D code.[4]

Spatial Metrics Updating Scheme

In the presence of small elastic deformations, the shape of $\Gamma_S(t)$ changes in time and therefore the spatial metrics used in the flow solver must be updated. For all grid points lying on the surface $\Gamma_S(t)$, $x'_{\xi'}(t)$ and $y'_{\xi'}(t)$ are deduced at each time step from the solution of the structural equation of motion Eq. (6). The departures of these derivatives from their initial values are denoted here by $(\Delta x'_{\xi'})_{i,1}$ and $(\Delta y'_{\xi'})_{i,1}$. The corresponding increments $(\Delta x'_{\xi'})$ and $(\Delta y'_{\xi'})$ for the grid points (i, j) external to $\Gamma_S(t)$ are derived from the former as follows:

$$(\Delta x'_{\xi'})_{i,j} = \frac{\bar{c}_{i,j}}{\bar{c}_{i,1}} \times \frac{r_{i,1}}{r_{i,j}} \times (\Delta x'_{\xi'})_{i,1}$$

$$(\Delta y'_{\xi'})_{i,j} = \frac{\bar{c}_{i,j}}{\bar{c}_{i,1}} \times \frac{r_{i,1}}{r_{i,j}} \times (\Delta y'_{\xi'})_{i,1} \tag{10}$$

where $\bar{c}_{i,j}$ is the distance between the grid points $(i-1, j)$ and $(i+1, j)$, and $r_{i,j}$ is the distance from the grid point (i, j) to the geometrical center of the structure, or the mid-chord in the case of an airfoil (see Fig. 2).

Validation and Application

Validation

A 128×40 two-dimensional O-grid is generated for the prediction of unsteady transonic flow around an NACA0012 airfoil in harmonic pitching. A two-spring undamped system is selected to model the structural dynamics behavior of the airfoil. For this problem, a single moving frame of reference located at the elastic center is capable of tracking the structural motion. The airfoil is forced into an oscillation around an axis located at the quarter-chord. The angle of attack is specified as

$$\alpha(t) = \alpha_m + \alpha_o \sin(\omega_f t) \tag{11}$$

where α_m is the mean angle of attack, α_o is the forced oscillation amplitude, and ω_f is the forced oscillation frequency. Using a dimensionless time unit, the above equation is written as

$$\alpha(t^*) = \alpha_m + \alpha_o \sin(2M_\infty k t^*) \tag{12}$$

where M_∞ is the freestream Mach number, $k = \omega_f c / 2U_\infty$ is the reduced frequency, c is the airfoil chord length, and t^* is the dimensionless time. The aeroelastic computations are carried out with $M_\infty = 0.755$, $\alpha_m = 0.016$ deg, $\alpha_o = 2.51$ deg, $k = 0.0814$ and a dimensionless time step $\Delta t^* = 0.015$.

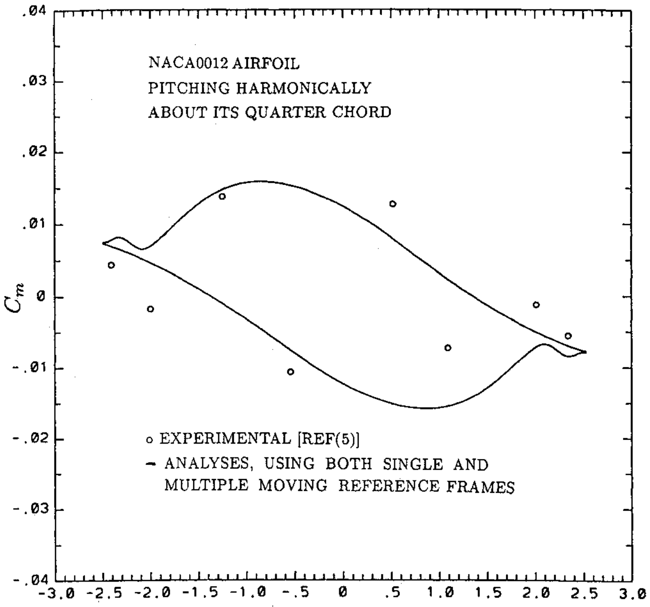

Fig. 3 C_m, pitching-moment coefficient variation during third cycle of sinusoidal pitching motion.

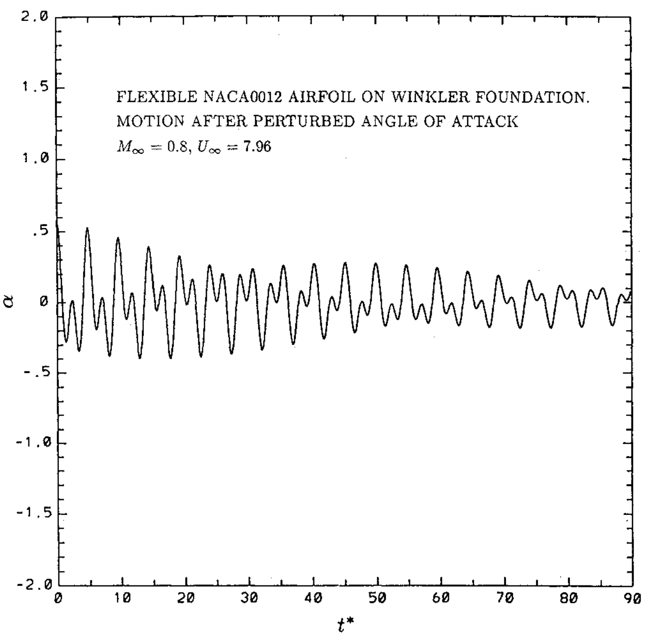

Fig. 4 Aeroelastic response of an infinite wing-like flexible structure.

The computed variations of the pitching-moment coefficient during the third cycle are depicted in Fig. 3. The solutions using one and multiple moving frames of reference are identical and overplot in the figure. More important, the results are shown to be in good agreement with the experimental data of Landon.[5]

Application

Next we consider the transient aeroelastic response of a wing-like structure in transonic flow. The structure is assumed to be indefinitely long so that a two-dimensional analysis can be performed. It includes a thin aluminum skin (Young modulus $E = 1.3 \times 10^7$) that is stiffened with three vertical and parallel rigid shear panels. An NACA0012 cross section is considered. This wing-like structure is assumed to rest on a Winkler-type foundation. The surface of the structure is discretized using two-node hermitian beam elements. Multiple moving frames of reference are invoked to follow the motion and track the deformation of the structural skin. The structure is placed in a flow at Mach number $M_\infty = 0.8$. The imposed velocity is $U_\infty = 7.96$. The angle of attack is slightly perturbed from an initial position and the response of the structure is computed. Figure 4 reports the variation of the angle of attack in time.

Future Work

The extension of this work to the three-dimensional case is in progress. The resulting code will be applied to the investigation of three-dimensional wing-body configurations. Benchmarks with alternative approaches will also be performed.

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Jet Mixing Enhancement by Hydrodynamic Excitation

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Introduction

IT is well established that acoustic excitation can increase mixing in jets. Previous work[1] (and references cited therein) has produced notable improvements in understanding the mixing of unheated jets at high and low speeds and of heated jets at low speed. At high Mach numbers, the mixing of heated jets was not significantly improved by acoustic excitation. One of these studies suggests that higher excitation levels may be needed to control high Mach number jets at high temperatures.

The excitation frequency must fall within a preferred range to be effective. The preferred frequency f is defined by a Strouhal number fD/U based on jet diameter D and velocity U. The preferred Strouhal number range of 0.2–0.5 is shown in Ref. 2.

The model consisted of a coannular jet system with provisions for holding rings in the secondary (annular or outer) flow stream as shown in Fig. 1. The primary nozzle is 7.5 cm (2.95 in.) in diameter and the secondary nozzle is 11 cm (4.32

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